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SOME ARGUMENTS FOR THE USE OF THE NOTION OF CONSTRUCTION

The main aim of logic is, as Frege has already declared, to guarantee entailment. For this aim, the analysis of sentences making up an argument is necessary. Thereby the logical analysis of natural language obeys the challenge of philosophical logic which is to determine the class of correct arguments.

Now consider the following case of an argument that contains propositional attitudes:

premise 1: *Mr. X knows that 2+3=5.*
premise 2: *2+3 = √25.* (It is not necessary to put this premise here.)
conclusion: *Mr. X knows that √25 is 5.*

Another argument containing notional attitudes:

premise: *Mr. X calculates 2+3.*
conclusion: *Mr. X calculates √25.*

Because of the analyticity of mathematics, any true mathematical sentence could be substituted for 2+3=5 in our first premise, truth-conditions will not change. In the case of the latter argument any expression denoting the same number can be substituted there. Now the consequent *conclusion*, however, is *not correct*: Mr. X has to know what he possibly does not know, or, in the second argument, he calculates what possibly he does not calculate. The problem concerned with our first example is called the *paradox of omniscience* and it is a special kind of the paradox of analysis.

By the way, remember that Frege tried to solve the *paradox of analysis* via his category of Sinn (sense) – premise 1: Mr. X knows that /E/vening star = /E/vening star, premise 2: /M/orning star = /E/vening star, conclusion: Mr. X knows that /M/orning star = /E/vening star; Frege said that in similar contexts, the proper names like /M/orning star do not denote directly the individual Venus but denotes only its senses – nevertheless the problematic consequence of Frege's claim is contextualism, the systematical ambiguity of nearly every word (and sentences as well). Note also that Frege did not avoid the paradox of omniscience: "It may happen, however, that the sense of the subsidiary clause is a complete thought, in which case it can be replaced by another of the same truth value without harm to the truth of the whole – provided there are no grammatical obstacles.” (Frege 1993, p. 39).

As is easy to see, the expression √25 *does not have the same meaning* as the expression 2+3 (a similar difference is also seen between expressions ‘equilateral triangle’ and ‘equiangular triangle’). That is why we cannot substitute mathematical expressions for each other in the above arguments. This problem, however, concerns not only mathematical expressions. For example any proposition can be constructed in infinitely many ways (imagine, for example, adding couple of negations). We have to take into account *the structure hidden behind the expression*.[1] The requirement of some structured entity is not entirely unknown: the *intensional isomorphism* of Rudolph Carnap (1958) or *structured meanings* introduced by Max Cresswell (1985) are examples thereof. In the contemporary logical semantics speaks about “*hyperintensional contexts*”, in the respective relations are called in transparent intensional logic “*constructional attitudes*”.

Now we will argue against two (inadequate) solutions to this problem.

a) Some people might think that calculating or drawing conclusions in such arguments like those above relates individuals with values (outcomes) of the given procedures. Namely when Mr. X is calculating 2+3 he is related to the number 5. We object that we often calculate something (perhaps complicated) without knowing what the results will be. Some mathematical problems can be calculated although nearly nobody is related to the respective outcome. Remember also that in a school we are not taught to know values, we are taught to calculate some mathematical task. We are taught to apply, for example, the addition function to the two numbers and then generate, or calculate, the outcome. So calculating is different from being related to, or strictly speaking aiming at, some outcome. Hence when Mr. X calculates 2+3, he is not related to the value.

To enlighten our topic from the other side we must also claim another natural fact. Mathematicians state equalities like 2+3=√25. This is their job, not to claim identities like 5=5. Both expressions, 2+3 and √25 denote the same object, number 5, but the ways, intellectual journeys, procedures, or computation processes are different; the number five is constructed by two distinct constructions.

1 For brilliant argumentation against semantical analysis that favours denoted objects as a base for substitutions let us cite Tichý (1988, pp. 234–5) (set aside here the Russellian “structured propositions”): “The *proposition* that 1+1=2 is the very same as the *proposition* that 2=1+1: it is the (unique) proposition which is true in all worlds at all times. When we say ‘1+1=2; therefore, 2=1+1,’ we are hardly relating this unstructured proposition to itself. Rather, we are relating two *compound* entities in which the same items are knitted together in two different ways. ... We have already noted that the proposition that 1+1=2 is the very same as the one which states the Pythagorean Theorem. Hence the former entails the latter. Yet one cannot *infer* the letter from the former: surely no one would argue that the square over the hypotenuse is the sum of the squares over the other two sides *because* one plus one makes two. For one mathematical truth to be inferable from another, the two must exhibit a certain *structural* affinity, an affinity which subsists between 1+1=2 and 2=1+1, but fails to subsist between 1+1=2 and the Pythagorean Theorem. Mathematical truths thus cannot be unstructured propositions. ... Inference is thus best seen as an operation on propositional *constructions* rather than on propositions.”

Briefly, *the calculating does not concern the outcome of a procedure: it concerns the procedure itself*. The calculating links an individual with constructions. From another point of view, for analysis of mathematical expressions, we have to postulate the category of some (structured) procedures.

b) According to another very popular opinion, calculating relates individuals with expressions. But immediately we have to object that it is entirely absurd to claim that by the statement "Mr. X calculates 2+3" is meant the same that can be – when using quotation marks for signing out the expression as such – written in this way: " Mr. X calculates (this verb is entirely meaningless in this context) '2+3'". (Let us add that in the sentence "Mr. X writes 2+3" we cannot substitute for the (sub)expression 2+3.) We must strictly separate an expression and what the expression stands for. A linguistic expression is always about an object distinct from this expression (of course, 'word' stands for word, ''word'' stands for 'word').[2]

It is also easy to see that calculating is independent of specific notation. It transcends the particular expressions. It is irrelevant whether Mr. X calculates 2+3, or two plus three, or, evoking Carnap, plus (II, III). If it is said that Mr. X calculates two plus three, we know nothing (and also do not care) about his acquaintance with standard or alternative arithmetical notation. The logician using Polish notation and the logician using the Russellian one are doing the same propositional calculus although their notations differ.

Hence *calculating concerns what the expressions mean rather than the expressions themselves*. When calculating, we are performing an arithmetical not a linguistic operation.[3] Analogously, in the propositional attitudes we are not related to the sentence-expression but to the meaning of this sentence (however not directly to the proposition, only to the construction of this proposition).

Now when the desirable structures are neither outcomes (results of some procedures), nor expressions as well, there is a question of what they are. What kind of entities are they? We know that it should be something what will distinguish the parts of the whole. Something that will contradistinguish 2, 3, + from 25, √ in the wholes like 2+3 and √25. But this is not, as we will see later, enough. Also we need something that will disclose such differences as those between 1+2 and 2+1

2 At this occasion we have to add an remark concerning constructions. In the sentence "[λx [^{0}> x ^{0}0]] constructs the class of positive numbers" we are talking about some relation between the construction and the predicate "construct the class of...", not about the expression of construction itself. About the expression '[λx [^{0}> x ^{0}0]]' we can correctly predicate only something like "contains two pairs of brackets". Constructions are not expressions, the expressions stand for constructions.

3 In connection with this note what combines objects (like 2, 3, +) cannot be an expression. A function adheres to its argument another way. Again cite Tichý here (1988, pp. 36–37): "If the term '(2 × 2) – 3' is not diagrammatic of anything, in other words, if the numbers and functions mentioned in this term do not themselves combine into any whole, then the term is the only thing which holds them together. The numbers and functions hang from it like Christmas decorations from a branch. The term, the linguistic expression, thus becomes more than a way of *referring* to independently specifiable subject matter: it becomes *constitutive* of it.".

where + is applied to the couple <1, 2> in the former case and to the couple <2, 1> in the latter (there might be people who have trouble with the fact that 2+1 derives the same number as 1+2). There are three possibilities on how to construe this structure. We will briefly argue against the first two views in favour of the third.

a) The simplest view is the opinion that such a structure can be rendered by sets. But Bernard Bolzano already noted, in his field of interest, that two concepts, for example "the learned son of the non-learned father" and "non-learned son of the learned father", can have the same the content, viz. the set {non, son, father, learned}, but that is not enough for the characteristics of concepts. The set is nothing other than the list of parts. The structure, i.e., the way the parts are combined together, is not visible from it. Computing 2+3 is not making the list of 2, +, 3 (or 2, 3, +, or +, 2, 3 etc.). *Sets (collections, sums) cannot in principle serve as the instrument for fixing the structure* which combines parts into the whole.

b) Another suggestion came from Cresswell who realized the necessity of analyzing mathematical expressions (and all hyperintensional contexts as well). He represented them as n-tuples of mathematical objects. So the structure 2+3 is construed as the triple < +, 2, 3 >. But also in this case we have the mere list, enumeration, of parts, although ordered. The n-tuple does not combine the enumerated objects into any whole. A convention is needed which interprets the triple as a proxy for the construction of the applying the first component (viz. the addition mapping) to the other two as arguments.[4] The triple is thus at best something what represents the structure that somebody is related to. (Another problem arose in cases like "Mr. X computes $[\lambda x[x{+}3]](2) = 5$". In Cresswell's theory, attitudes are between someone and mathematical object, but in Cresswell's hierarchy of objects there is nothing like a variable in an objectual sense. Thus there is no way to represent the meaning of clauses like this.) Therefore also *n-tuples are not plausible tools for fixing the required structure*.[5]

c) As an argument for the choice of apparatus, let us consider following intuition (adapting an example with which Tichý started his main book, Tichý 1988). Surely there are many ways to Rome. Someone who decided to visit Rome can get there by plane from some city, someone can go there by bus through some cities, somebody through some other cities. The ways and the places somebody goes through differ from each other. The journeys to Rome (this is the destination) have different itineraries. One's *destination* and the *itinerary* one follows to get there are clearly two *distinct items*. Whereas Rome is simply some place, an itinerary is a *compound* in which a number of locations occur. This intuition has

4 Carnap tried to define his intensional isomorfism also without references to the some notion of construction of the whole. Expressions '8–3' and 'minus(VIII, III)' are, according to his definition, intensionally isomorph. It can be argued that his definition presupposes only "regimented" languages where is a perfect isomorfism between a calculation and the formula which represents it and left-right convention is used (ordinary English is excluded, consider, for example, an expression like 'subtracting 3 from 8'). But what is worse, left-to-right convention of writing arguments must be explicitly stated. The convention concerns the method of recording the application of a binary function to two arguments and this is a symbolising a certain calculation (construction). (See the first pages of Tichý 1986 and Tichý 1988, pp. 8–9.)

5 For a more precise argumentation see Tichý (1994b, pp. 78–80) or Cmorej & Tichý (1988).

a clear correlation in the field of mathematics. An arithmetical calculation demonstrated by the expression is much like the itinerary, *an intellectual journey* whose destination is some number. The number 5 we can get through adding 3 to 2 or the square root of 25 (and so on). None of these intellectual journeys is a part of the number 5. Hence the equality 5=5 is not interesting; what is interesting is the equality $2+3=\sqrt{25}$ which says that two intellectual journeys, procedures have the same destination. We have seen above that a (possibly ordered) list of items is not sufficient. The journey to Rome consists not only of some cities and the means of transport, but in their combining of the whole of journey. The numbers 2, 3, and the + must be clearly combined into a certain compound.

We are looking for such an apparatus where the role of parts is clearly defined. We can find such a formal tool in Church's-calculi, more precisely in *typed lambda calculi*. Lambda-calculi construe functions not as a mappings, i.e., a simple correspondence between arguments and values, but as some calculating methods. The difference between mappings and functions as calculating methods is easily seen due to the fact that two mappings effecting one and the same correspondence are one and the same mapping (extensionality). Two calculating methods, however, may be distinct although they yield the same correspondence between arguments and values: the values of one and the same mapping are obtainable from its arguments by more than one (strictly speaking infinitely many) calculating methods. That is why the notion of calculating method, *the calculating procedure* is so interesting for us. There are (infinitely) many constructions of one proposition (analogously for other objects). We have seen in the example of hyperintensional contexts that the view that when calculating we are related to the values of procedures is inadequate. The other point concerns the structure we need. Whereas the table (of a mapping) is unstructured, the calculating procedure is structured. Note that originally, in the beginning of seventeenth century, mathematicians considered functions as functional prescriptions (which are structured), not as bare correspondences. A lambda calculus is an apparatus that recognises three kinds of those procedures: variables (which compute dependently on valuation), application (which computes the value of a function on its argument), and abstraction (which produces the function itself). So 2+3 is an example of application of addition to two and three ([+ 2 3]), $\lambda x[+\ [\times\ 2\ x]\ 3]$ is an example of abstraction that determines a function which associates any number with the outcome of multiplying it by 2 and adding 3. The role of parts is given by rigorous definition, for example in the case of application addition mapping is a binary function of couples of numbers and the numbers 2, 3 make up just such argument.

Tichý adapted lambda-calculus; he modified it into his theory of *constructions*.[6] There are four main kinds of constructions. First, the *variables* which construct dependent on valuation (variables are construed in an objectual way). Then for

6 As an remark we add that constructions, as well as numbers and any other abstract entities are not spatio-temporally localizable (they are, e.g., not expressions), they are abstract. If you are asking where and when constructions are, you can liken them to numbers for which this question does not have any reasonable sense.

the identification, the immediate, one-step ”grasp” of some object we have *trivialization*, that construct X without any change (trivialization is an explication of “primitive sense”). Application is turned to *composition* which after the identification of entities (their constructing by trivialization) applies a function to arguments. Abstraction is turned into *closure* which is a certain construction of some function. Constructions fix particular steps of procedures, parts are not lost as in the table of mapping. Parts are clearly visible and their role in composing the whole is given by rigorous definition.

Constructions are also an explication of Frege’s Sinn (“sense“, “mode of presentation”). Frege’s example with medians of a triangle leads, in contrast to his morning star – evening star example, to our claim about the structuredness of sense. The expressions ‘equiangular triangle’ and ‘equilateral triangle’ depict two different constructions (because of different items) of one and the same (denoted) object. It can be easily seen also from such simple examples of expressions like 2+3 and √25 that it is not important to know the denoted object, but rather the structured procedure. To *understand* some expression is to know which construction the expression represents. A sentence is about the constituents of the construction it expresses. To *translate* some expression into another language means to represent the same construction, the same intellectual journey from some given objects to another.[7]

And what is more, we cannot avoid constructions in the conception of such fundamental notion as that of fact. The expressions “Alan is taller than Bill” and “Bill is shorter than Alan” denote one and the same fact. But the two sentences say it differently: the former is about applying taller-than relation (or function) to Alan and Bill, the latter about applying shorter-than relation to Bill and Alan. The proposition, fact, is only one, it does not contain parts like the taller-than relation or shorter-than relation. It is thus a sentence that is a picture of the construction of some fact, state of affairs.[8]

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7 For Tichý logic is not reducible to a set of formal tools. If it is an tool then for inquiring entailment and for this purpose also for analysing expressions ingoing into the entailment: “*Logic is the study of logical objects* (individuals, truth-values, possible worlds, propositions, classes, properties, relations, and the like) *and the ways such objects can be constructed from other such objects*. ... The point of investigating logical constructions of objects is two-fold. In the first place, *the nature of such constructions often guarantees noteworthy properties or relationships between objects generated by those constructions*. ... In the second place, *logical constructions can be assigned to linguistic expressions as their analyses* (emphasis mine; Tichý 1978, p. 275).

8 The very common ascribing of the structure of propositional constructions to propositions themselves does not end by this. Propositions are often called negative (existential, disjunctive, etc.). But note that every proposition can be constructed by negating another proposition. This is, however, some interesting claim about the particular construction of the proposition, not about the proposition itself. See also Tichý 1986 (part 1), 1988 (pp. 14–15), or 1994a (an analysis of Wittgenstein’s “picture theory”), or 1995.

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NĚKTERÉ DŮVODY PRO UŽITÍ POJMU KONSTRUKCE

K prověření logického vyplývání a pro adekvátní logickou sémantiku přirozeného jazyka se jeví nezbytné přijmout nejen intenzionální, ale také hyperintenzionální entity. Jak ukazují analýzy propozičních postojů (a logika postojů vůbec), vhodným kandidátem pro takovýto druh entit jsou – spíše než Cresswellovy uspořádané n-tice – konstrukce, jak jsou definovány v transparentní intenzionální logice P. Tichého.